P-wave contribution to three-body B decays

Hsiang-nan Li
Academia Sinica, Taipei

Presented at High 1

Feb. 07, 2017

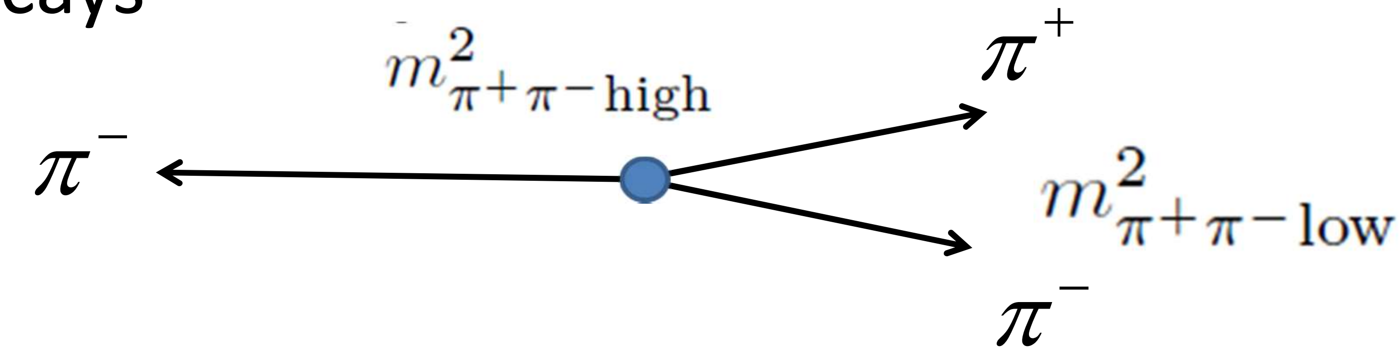
Outlines

- Introduction
- Two-hadron distribution amplitudes
- P-wave contribution
- Summary

Introduction

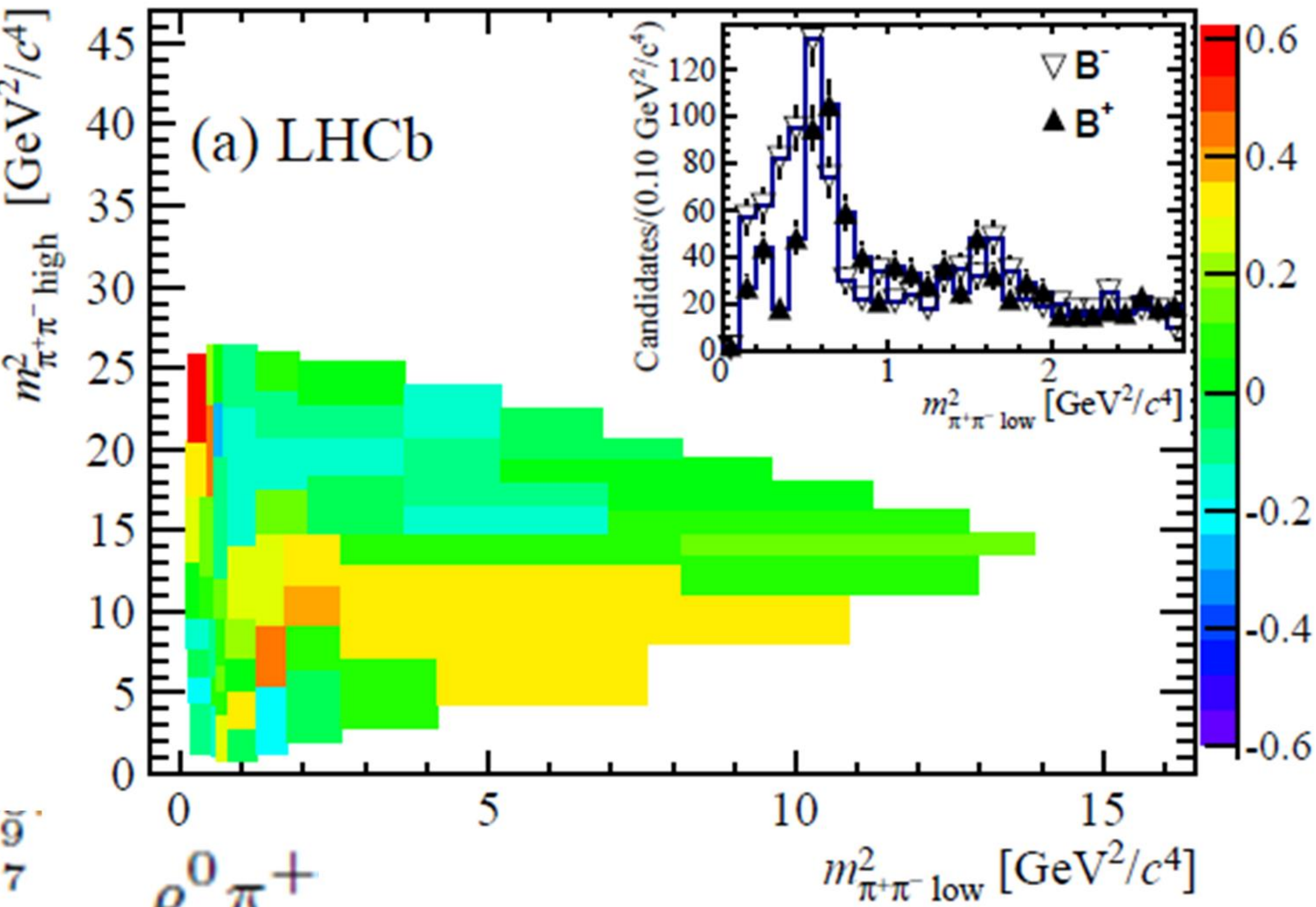
3-body hadronic B decays

- 3-body B decays more complicated than 2-body, including both resonant contribution $B \to \pi\rho \to \pi\pi\pi$ and nonresonant one, containing stronger final-state interaction (FSI)
- Definitions of invariant masses in 3-body B decays



Dalitz plot

- LHCb has measured CP asymmetries in whole Dalitz plot



$0.18^{+0.09}_{-0.17}$ $\rho^0\pi^+$

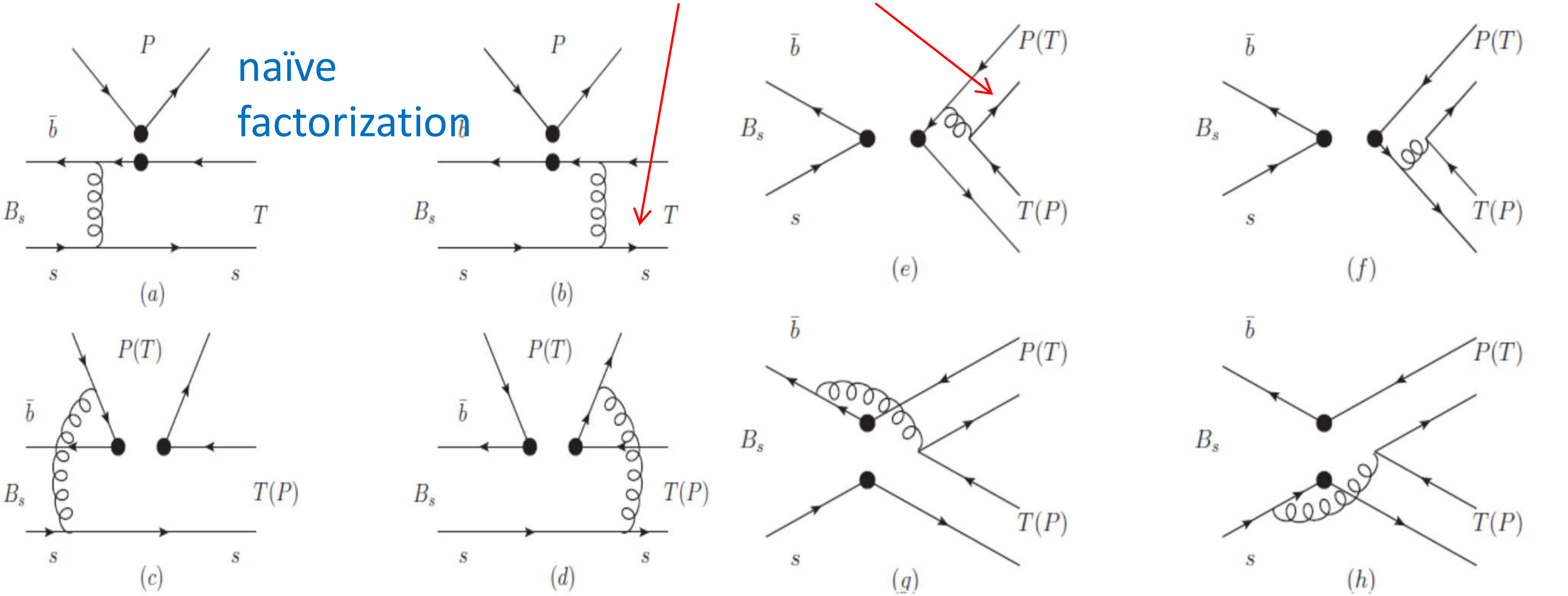
Motivation

- Data result from entangled nonresonant and resonant contributions, and of different partial waves
- Develop a theoretical approach to 3-body hadronic B decays
- Understand data and predict direct CP asymmetries of 3-body decays in localized regions of phase space
- Very challenging!

PQCD for 2-body B decays

- PQCD approach to 2-body B decays based on kT factorization: b quark decay kernel convoluted with TMD hadron wave functions
- Parton kT smears end-point singularity

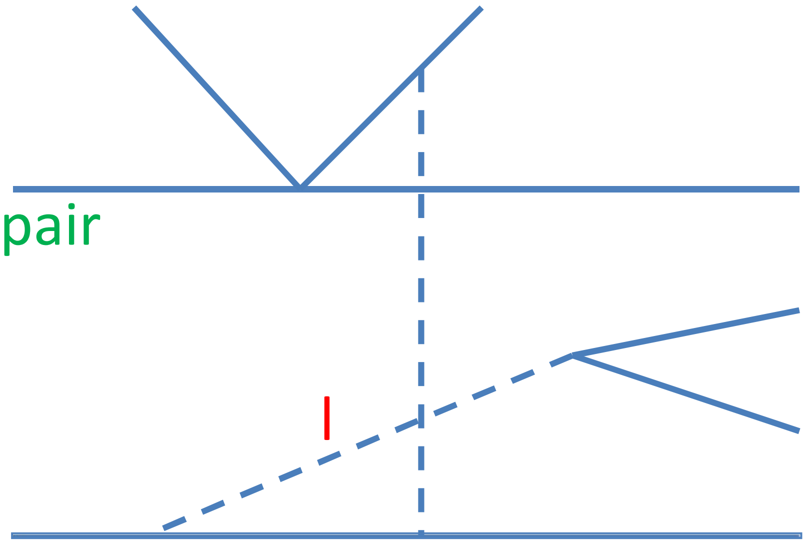
naïve factorization



Typical diagram for 3-body decay

partial counting
8 X 2 X 8 = 128

attachment of l
location of pion pair
LO diagrams



Invariant mass
of pion pair w

$$l^2 \sim w^2$$

$w^2 \sim m_B^2$ power suppressed compared to

$$w^2 \sim \Lambda m_B, \Lambda^2$$

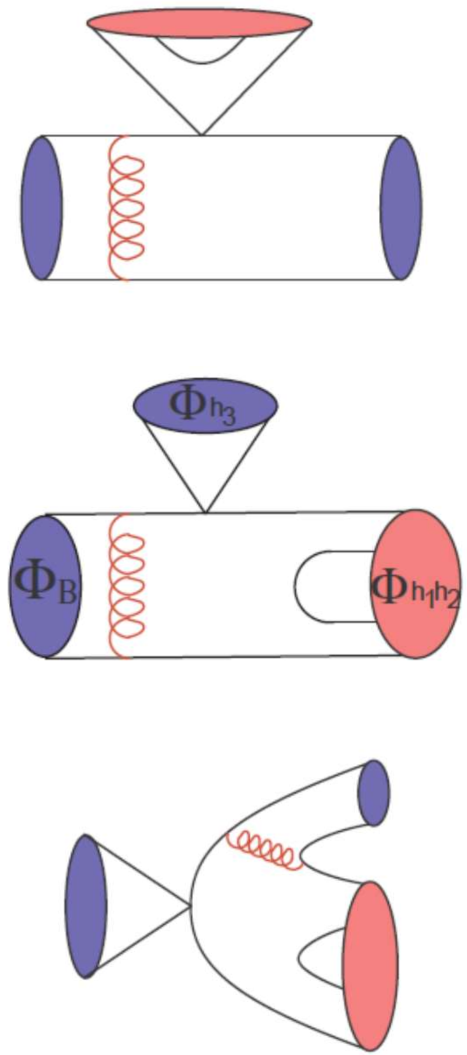
Approaches in literature

- Based on parameterizations of

  current-induced process

  transition process

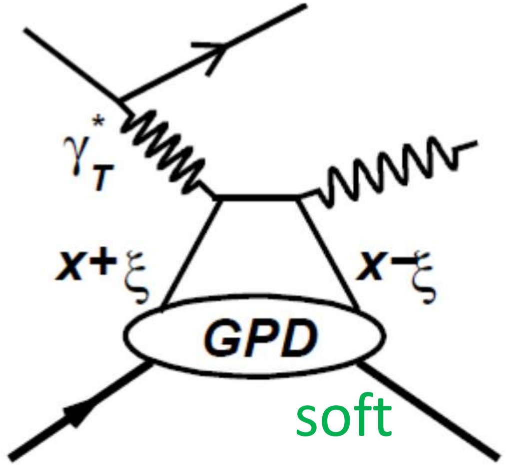
  But, annihilation process?
- Nonfactorizable contribution?
- Resonant via Breit-Wigner then double counting of nonresonant?
- Rescattering (FSI) strong phases?



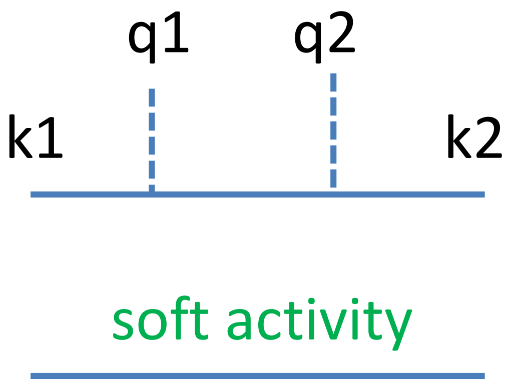
Two-hadron distribution amplitudes

Our proposal in 2002 (Chen, Li)

- Inspired by generalized parton distribution (GPD) based on dominance of hand-bag diagram in forward scattering



- Non-forward, same order of magnitude



k1 // k2
no need of hard gluon

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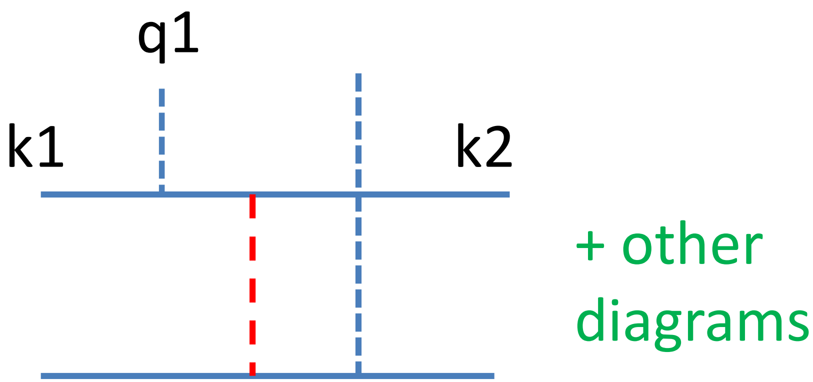
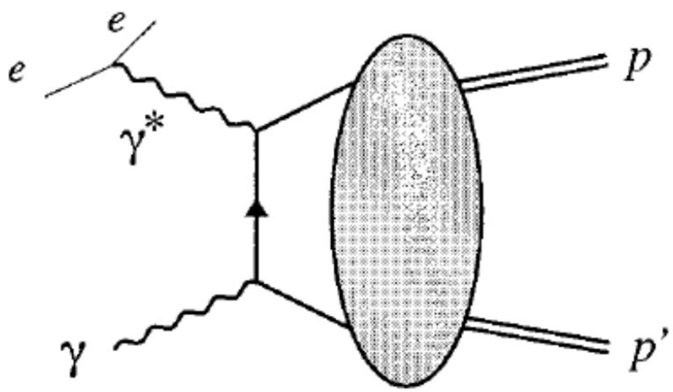
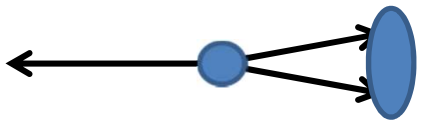


k1+q1 = k2? k2 off-shell
need hard gluon

Two-hadron DA

- Introduce two-hadron distribution amplitude (TDA, crossing of GPD) for dominant region in 3-body B decays





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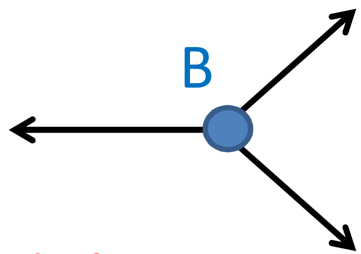

one hard, one soft dominant as two hadrons collimate

two hard gluons power suppressed

3-body reduced to 2-body

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Physics Letters B 561 (2003) 258–265

www.elsevier.com/locate/npe

Three-body nonleptonic B decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li

Institute of Physics, Academia Sinica, Taipei 115, Taiwan, ROC

Received 18 March 2003; accepted 24 March 2003

Editor: T. Yanagida

Abstract

We develop perturbative QCD formalism for three-body nonleptonic B meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

Definitions of TDAs

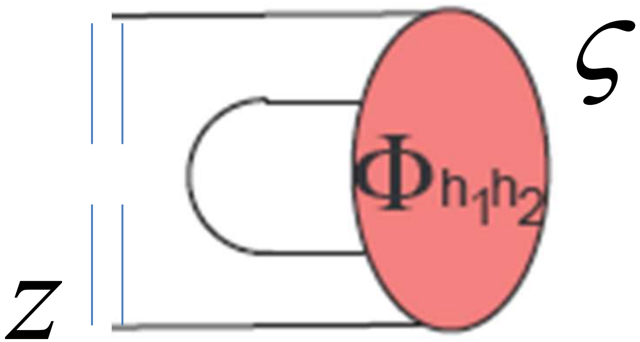
- TDAs for vector, scalar, tensor currents (from Fierz transformation for factorizing quark flow)

$$\Phi_v(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-)\ \not{n}_- T\psi(0)|0\rangle\ ,$$

$$\Phi_s(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \frac{P^+}{w} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-) T\psi(0)|0\rangle\ ,$$

$$\Phi_t(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \frac{f_{2\pi}^{\perp}}{w^2} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-) i\sigma_{\mu\nu} n_-^{\mu} P^{\nu} T\psi(0)|0\rangle$$

$$p = p_1 + p_2 \qquad \omega^2 = p^2$$



$T = \sigma^3/2,\quad I = 1$ Isovector, P-wave

$T = 1/2,\quad I = 0$ Isosinglet, S-wave

Parameterization of TDAs

$$\propto (p_1 - p_2)^\mu F_\pi$$

- Normalization

$$\int_0^1 dz \Phi_{\parallel}^{I=1}(z,\zeta,w^2) = (2\zeta - 1) F_\pi(w^2)$$

$$\int_0^1 dz \Phi_{\perp}^{I=1}(z,\zeta,w^2) = (2\zeta - 1) F_t(w^2)$$

- Up to leading partial wave expansion

$$\Phi_{v,t}(z,\zeta,w^2) = \frac{3F_{\pi,t}(w^2)}{\sqrt{2N_c}} z(1-z)(2\zeta - 1)$$

complex time-like from data
include FSI

correspond to l = 1
P wave…

$$\Phi_s(z,\zeta,w^2) = \frac{3F_s(w^2)}{\sqrt{2N_c}}$$

correspond to l = 0
S wave…

form factors Fs, Ft, twist-3,
suppressed by a power in PQCD

Wang, Hu, Li, Lu 2014

P-wave contribution

Wang, Li 2016

Motivation

- M. Nakao asked about P-wave contribution during 2015 Winter Conference at High-1, where I talked about S Wave
- For complete analysis for Dalitz plots, we do need inputs of P-wave two-hadron DAs
- Consider quasi-two body $B \to K\rho \to K\pi\pi$ for which BaBar, Belle, LHCb data are available
- Can also check consistency with two-body PQCD formalism for $B \to K\rho$

Parameterization

- P-wave two-pion Das

Gegenbauer moments to be determined

$$\phi^{I=1}_{v\nu=-}(z,\zeta,w^2) \equiv \phi^0(z,\zeta,w^2) = \frac{3F_\pi(w^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a_2^0 C_2^{3/2}(1-2z)\right] P_1(2\zeta-1)\,,$$

$$\phi^{I=1}_{s}(z,\zeta,w^2) \equiv \phi^s(z,\zeta,w^2) = \frac{3F_s(w^2)}{2\sqrt{2N_c}} (1-2z)\left[1 + a_2^s\left(1-10z+10z^2\right)\right] P_1(2\zeta-1)$$

$$\phi^{I=1}_{t\nu=+}(z,\zeta,w^2) \equiv \phi^t(z,\zeta,w^2) = \frac{3F_t(w^2)}{2\sqrt{2N_c}} (1-2z)^2\left[1 + a_2^t C_2^{3/2}(1-2z)\right] P_1(2\zeta-1)\,,$$

- Form factor input from e+e- annihilation data

$\rho-\omega$ mixing

BaBar 2012

$$F_\pi(w^2) = \left[\mathrm{GS}_\rho(w^2, m_\rho, \Gamma_\rho)\frac{1 + c_\omega \mathrm{BW}_\omega(w^2, m_\omega, \Gamma_\omega)}{1 + c_\omega} + \sum c_i \mathrm{GS}_i(w^2, m_i, \Gamma_i)\right]\left(1 + \sum c_i\right)^{-1}$$

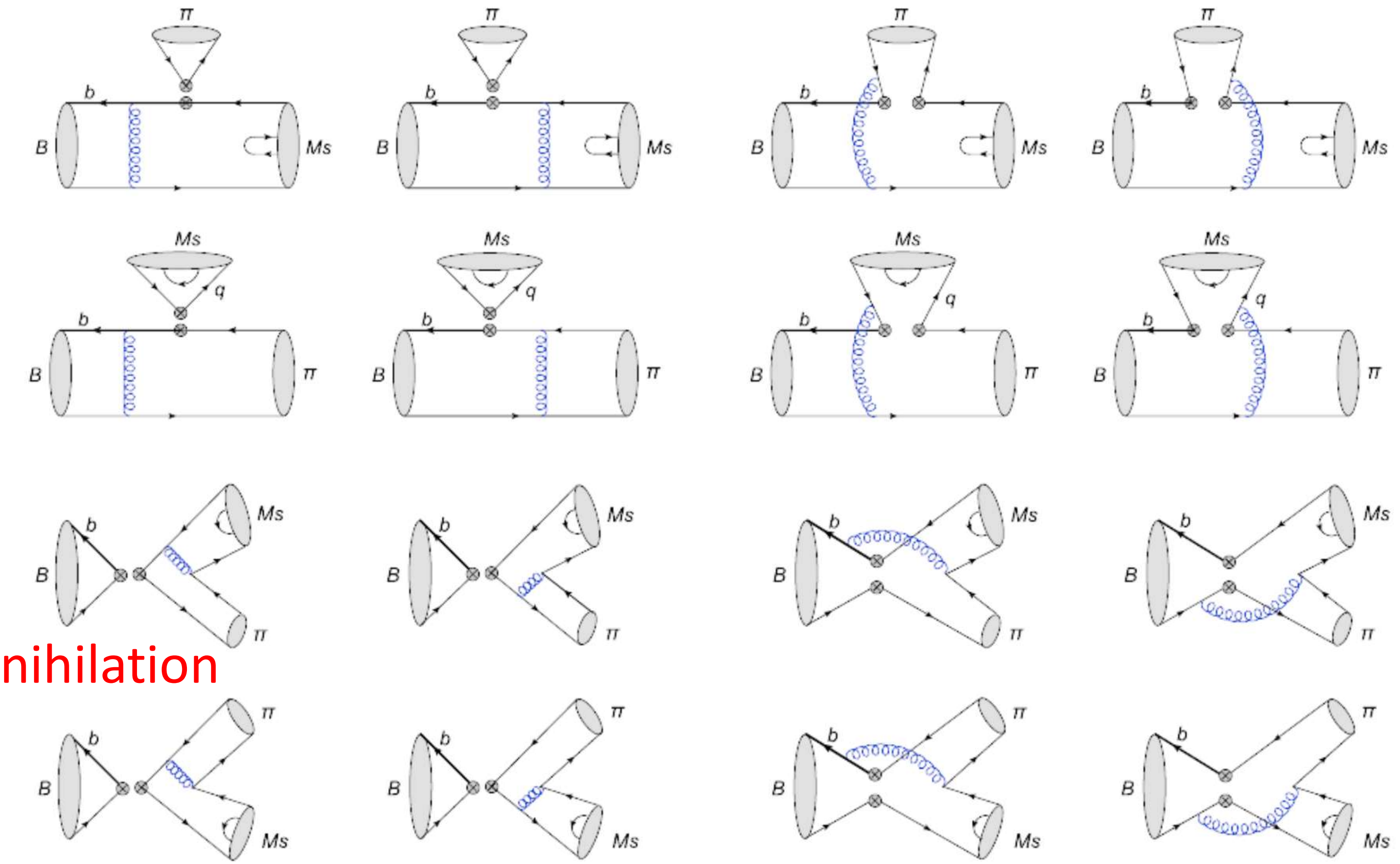
Gounaris-Sakurai model

Breit-Wigner function

$$i = \rho'(1450),\ \rho''(1700)\ \text{and}\ \rho'''(2254)$$

Feynman diagrams

- All inputs are ready, go ahead to calculate 16 diagrams (similar to 2-body decays) nonfactorizable



annihilation

Results

- Fitted P-wave Gegenbauer moments from $K\rho$ channels

$$a_2^0 = 0.25,\ a_2^s = 0.75,\ \text{and}\ a_2^t = -0.60$$

		Results	Data [98]
$K^+\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$3.42^{+0.78}_{-0.55}(\omega_B)^{+0.44}_{-0.39}(a_2^t)^{+0.39}_{-0.38}(m_0^K)^{+0.39}_{-0.32}(a_2^0)^{+0.29}_{-0.28}(a_2^s)$	3.7 ± 0.5
	$\mathcal{A}_{CP}$	$0.43^{+0.04}_{-0.05}(\omega_B) \pm 0.06(a_2^t) \pm 0.03(m_0^K) \pm 0.03(a_2^0) \pm 0.01(a_2^s)$	0.37 ± 0.10
$K^0\pi^+\pi^0$	$\mathcal{B}\ (10^{-6})$	$7.43^{+1.92}_{-1.31}(\omega_B)^{+1.65}_{-1.42}(a_2^t)^{+0.88}_{-0.91}(m_0^K)^{+0.60}_{-0.62}(a_2^0)^{+0.53}_{-0.47}(a_2^s)$	8.0 ± 1.5
	$\mathcal{A}_{CP}$	$0.15^{+0.02}_{-0.01}(\omega_B)^{+0.04}_{-0.05}(a_2^t) \pm 0.01(m_0^K)^{+0.01}_{-0.00}(a_2^0) \pm 0.00(a_2^s)$	-0.12 ± 0.17
$K^+\pi^-\pi^0$	$\mathcal{B}\ (10^{-6})$	$6.51^{+1.71}_{-1.12}(\omega_B)^{+0.58}_{-0.61}(a_2^t)^{+0.78}_{-0.77}(m_0^K)^{+0.67}_{-0.64}(a_2^0)^{+0.39}_{-0.47}(a_2^s)$	7.0 ± 0.9
	$\mathcal{A}_{CP}$	$0.31^{+0.00}_{-0.01}(\omega_B)^{+0.09}_{-0.08}(a_2^t)^{+0.03}_{-0.02}(m_0^K) \pm 0.01(a_2^0) \pm 0.02(a_2^s)$	0.20 ± 0.11
$K^0\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$3.76^{+1.09}_{-0.74}(\omega_B)^{+0.73}_{-0.60}(a_2^t)^{+0.52}_{-0.47}(m_0^K)^{+0.28}_{-0.25}(a_2^0)^{+0.26}_{-0.23}(a_2^s)$	4.7 ± 0.6
	$\mathcal{A}_{CP}$	$0.06^{+0.01}_{-0.02}(\omega_B)^{+0.00}_{-0.01}(a_2^t) \pm 0.00(m_0^K)^{+0.00}_{-0.01}(a_2^0) \pm 0.00(a_2^s)$	$-$

Consistency with 2-body formalism

- BRs are close between 3-body and 2-body formalism, direct CPAs differ a bit
- CPA from 3-body more consistent with data

$$K^+\rho^0 \quad \begin{cases} \mathcal{B} = (3.52^{+0.67}_{-0.45}(\omega_B)^{+0.40}_{-0.34}(a_2^t)^{+0.42}_{-0.38}(m_0^K)^{+0.47}_{-0.43}(a_2^0)^{+0.25}_{-0.24}(a_2^s)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.55^{+0.02}_{-0.04}(\omega_B)^{+0.09}_{-0.08}(a_2^t) \pm 0.03(m_0^K)^{+0.00}_{-0.01}(a_2^0) \pm 0.01(a_2^s)\,, \end{cases}$$

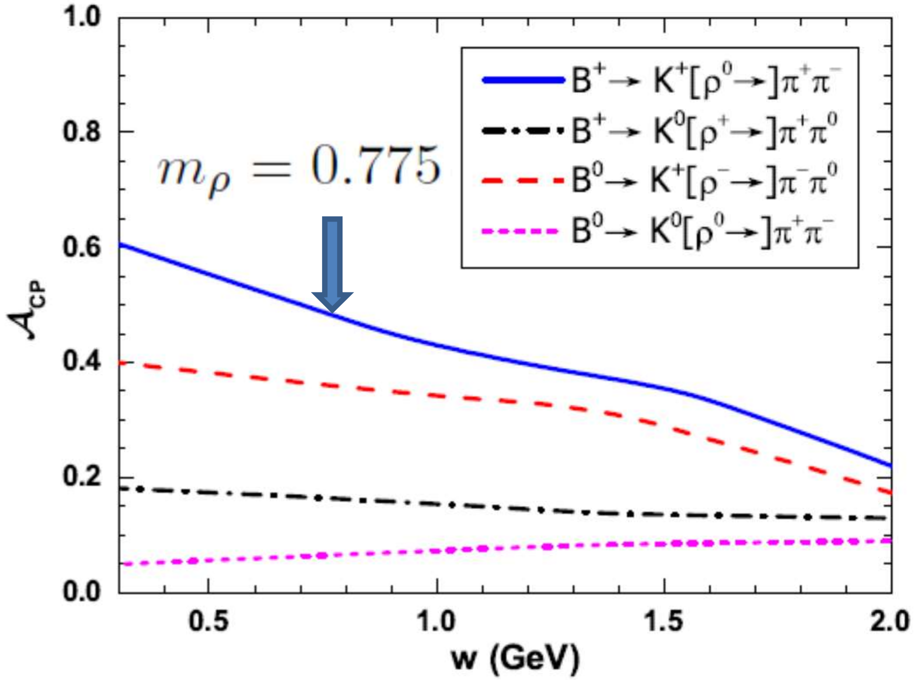
$$K^0\rho^+ \quad \begin{cases} \mathcal{B} = (7.66^{+1.79}_{-1.19}(\omega_B)^{+1.69}_{-1.44}(a_2^t)^{+1.04}_{-0.95}(m_0^K)^{+0.84}_{-0.73}(a_2^0)^{+0.43}_{-0.41}(a_2^s)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.22 \pm 0.03(\omega_B)^{+0.03}_{-0.05}(a_2^t) \pm 0.01(m_0^K) \pm 0.00(a_2^0) \pm 0.00(a_2^s) \end{cases}$$

$$K^+\rho^- \quad \begin{cases} \mathcal{B} = (6.92^{+1.58}_{-1.04}(\omega_B)^{+0.67}_{-0.53}(a_2^t)^{+0.86}_{-0.81}(m_0^K)^{+0.91}_{-0.80}(a_2^0)^{+0.42}_{-0.40}(a_2^s)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.34^{+0.00}_{-0.01}(\omega_B)^{+0.13}_{-0.12}(a_2^t)^{+0.03}_{-0.02}(m_0^K)^{+0.01}_{-0.02}(a_2^0)^{+0.01}_{-0.02}(a_2^s)\,. \end{cases}$$

$$K^0\rho^0 \quad \begin{cases} \mathcal{B} = (4.01^{+1.07}_{-0.71}(\omega_B)^{+0.70}_{-0.63}(a_2^t)^{+0.55}_{-0.50}(m_0^K)^{+0.40}_{-0.35}(a_2^0) \pm 0.19(a_2^s)) \times 10^{-6} \\ \mathcal{A}_{CP} = 0.04 \pm 0.01(\omega_B) \pm 0.00(a_2^t) \pm 0.00(m_0^K)^{+0.00}_{-0.01}(a_2^0) \pm 0.00(a_2^s) \end{cases}$$

More results

- Differential distribution of CPA
- CPA of $K^+\rho^0$ would be overestimated in 2-body formalism



- Predictions

$K^+\rho'^0 \to K^+\pi^+\pi^-$	$\mathcal{B}\ (10^{-7})$	$4.32^{+1.17}_{-0.99}(\omega_B)^{+0.81}_{-0.79}(a_2^t)^{+0.59}_{-0.64}(a_2^s)^{+0.40}_{-0.46}(m_0^K)^{+0.13}_{-0.17}(a_2^0)$
	$\mathcal{A}_{CP}$	$0.32^{+0.06}_{-0.04}(\omega_B) \pm 0.03(a_2^t)^{+0.01}_{-0.02}(a_2^s)^{+0.02}_{-0.01}(m_0^K) \pm 0.01(a_2^0)$
$K^0\rho'^+ \to K^0\pi^+\pi^0$	$\mathcal{B}\ (10^{-7})$	$10.37^{+3.72}_{-2.36}(\omega_B)^{+3.14}_{-2.71}(a_2^t)^{+1.26}_{-1.03}(a_2^s)^{+1.13}_{-0.92}(m_0^K)^{+0.42}_{-0.37}(a_2^0)$
	$\mathcal{A}_{CP}$	$0.12 \pm 0.02(\omega_B)^{+0.02}_{-0.01}(a_2^t)^{+0.03}_{-0.02}(a_2^s) \pm 0.01(m_0^K) \pm 0.01(a_2^0)$
$K^+\rho'^- \to K^+\pi^-\pi^0$	$\mathcal{B}\ (10^{-7})$	$7.61^{+2.37}_{-1.90}(\omega_B)^{+1.32}_{-1.03}(a_2^t)^{+1.17}_{-0.88}(a_2^s)^{+0.86}_{-0.75}(m_0^K)^{+0.26}_{-0.22}(a_2^0)$
	$\mathcal{A}_{CP}$	$0.27^{+0.02}_{-0.01}(\omega_B) \pm 0.06(a_2^t)^{+0.00}_{-0.01}(a_2^s) \pm 0.02(m_0^K) \pm 0.01(a_2^0)$
$K^0\rho'^0 \to K^0\pi^+\pi^-$	$\mathcal{B}\ (10^{-7})$	$4.84^{+1.82}_{-1.32}(\omega_B)^{+1.11}_{-1.05}(a_2^t) \pm 0.50(a_2^s)^{+0.48}_{-0.46}(m_0^K)^{+0.14}_{-0.16}(a_2^0)$
	$\mathcal{A}_{CP}$	$0.08^{+0.00}_{-0.01}(\omega_B)^{+0.02}_{-0.00}(a_2^t) \pm 0.01(a_2^s) \pm 0.01(m_0^K) \pm 0.01(a_2^0)$

Summary

- Systematic approach to 3-body B decays with two-hadron DAs has been established
- Can include both resonant and nonresonant contributions in time-like form factors
- Both S-wave and P-wave two-pion DAs have been determined
- Consistency between 3-body and 2-body PQCD formalisms has been verified
- Ready to explain and predict direct CPAs of 3-body B decays in various localized regions of phase space

Back-up slides

Kinematics

- Meson momenta in light-cone coordinates

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\rm T}), \quad p = \frac{m_B}{\sqrt{2}}(1, \eta, 0_{\rm T}), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1-\eta, 0_{\rm T})$$

- Two-hadron invariant mass

$$\omega^2 = p^2 \qquad p = \underbrace{p_1}_{\text{pi+}} + \underbrace{p_2}_{\text{pi-}} \qquad \eta = \frac{\omega^2}{m_B^2}$$

- pi+ momentum fraction

$$p_1^+ = \zeta\frac{m_B}{\sqrt{2}}, \quad p_1^- = (1-\zeta)\eta\frac{m_B}{\sqrt{2}}, \quad p_2^+ = (1-\zeta)\frac{m_B}{\sqrt{2}}, \quad p_2^- = \zeta\eta\frac{m_B}{\sqrt{2}}$$

pion momentum fraction

C-parity

- C-parity (charge parity) for mesonic state is equivalent to parity

$$\mathcal{C}\,|\pi^+\,\pi^-\rangle = (-1)^L\,|\pi^+\,\pi^-\rangle$$

- C-parity for quark fields (spinors)

$$\psi^{(c)} = C\psi^\star \qquad C = i\gamma^2$$

$$C^\dagger\gamma^\mu C = -(\gamma^\mu)^\star$$

- C-parity is odd for vector and tensor currents, and even for scalar current

Motivation

- Recent LHCb data of direct CP asymmetries in localized regions of phase space

$$A_{CP}^{\text{region}}(K^+K^-K^-) = -0.226 \pm 0.020 \pm 0.004 \pm 0.007,$$

for $m^2_{K^+K^-\text{high}} < 15\ \text{GeV}^2$ and $1.2 < m^2_{K^+K^-\text{low}} < 2.0\ \text{GeV}^2$

$$A_{CP}^{\text{region}}(K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.032 \pm 0.007$$

for $m^2_{K^-\pi^+\text{high}} < 15\ \text{GeV}^2$ and $\underline{0.08 < m^2_{\pi^+\pi^-\text{low}} < 0.66\ \text{GeV}^2}$

$$A_{CP}^{\text{region}}(K^+K^-\pi^-) = -0.648 \pm 0.070 \pm 0.013 \pm 0.007$$

for $m^2_{K^+K^-} < 1.5\ \text{GeV}^2$

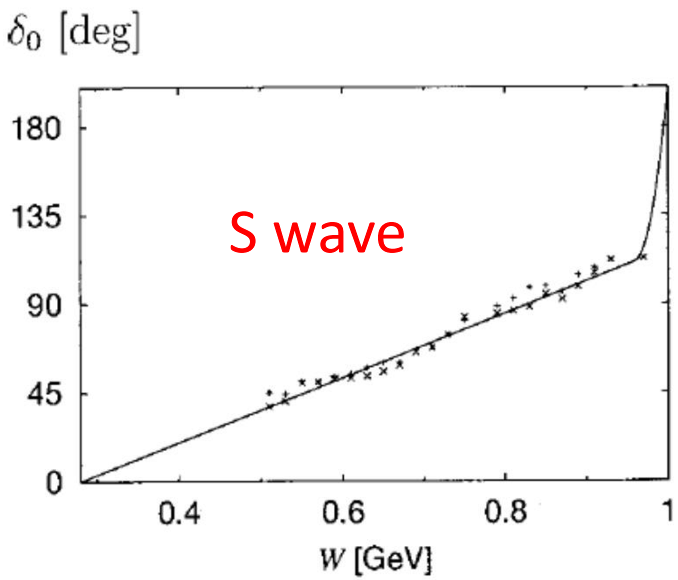
rho resonance

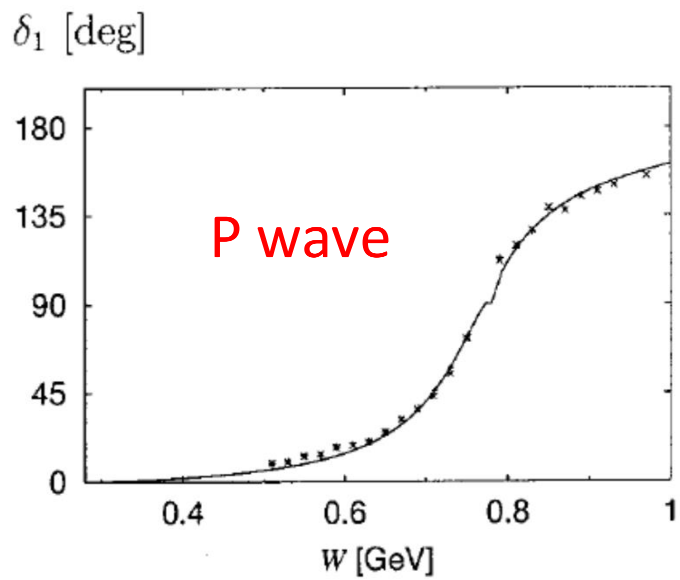
$$A_{CP}^{\text{region}}(\pi^+\pi^-\pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$

for $m^2_{\pi^+\pi^-\text{high}} > 15\ \text{GeV}^2$ and $\underline{m^2_{\pi^+\pi^-\text{low}} < 0.4\ \text{GeV}^2}$

Rescattering phases

- LHCb data of CP asymmetries in localized regions (nonresonant only) offered a chance to confront our theory
- Data for rescattering phases in localized region ($m_{\pi\pi}^2 < 0.4\, GeV^2$) are available





Direct CP asymmetry

- Factorization formula for decay amplitude

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}$$

- B meson and hadron DAs have been widely adopted in analysis of 2-body decay
- Calculate B+ and B- decays

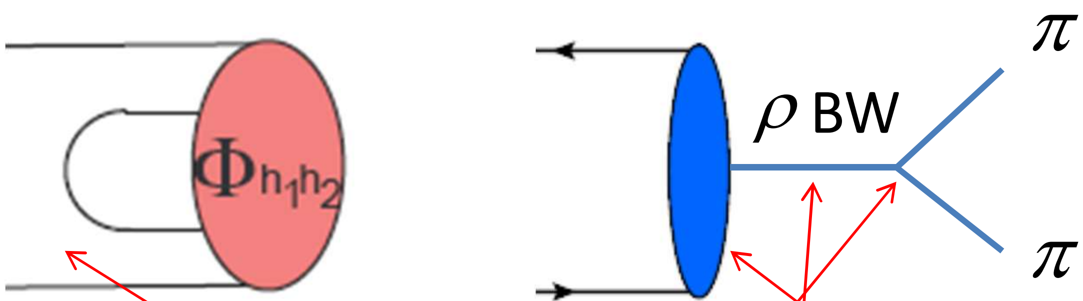
$$A_{CP}^{reg}(B^\pm \to \pi^\pm \pi^+ \pi^-) = 0.52\,^{+0.12}_{-0.22}(\omega_B)\,^{+0.11}_{-0.09}(a_2^\pi)\,^{+0.03}_{-0.03}(m_0^\pi)$$

+-0.05 +-0.15 +-0.1

- Data $A_{CP}^{\rm region}(\pi^+\pi^-\pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$
- Short-distance (annihilation) phase important
- P wave phase doubled, Acp increases up to 0.7

Form factor ratio

- Consistency between two-pion Das and pole model



$$F_\pi^\rho(w^2) \approx \frac{g_{\rho\pi\pi} w f_\rho}{D_\rho(w^2)}$$

- At twist 3

$$F_{s,t}^\rho(w^2) \approx g_{\rho\pi\pi} w f_\rho^T / D_\rho(w^2)$$

- Form factor ratio

$$F_{s,t}(w^2) \approx (f_\rho^T / f_\rho) F_\pi(w^2)$$

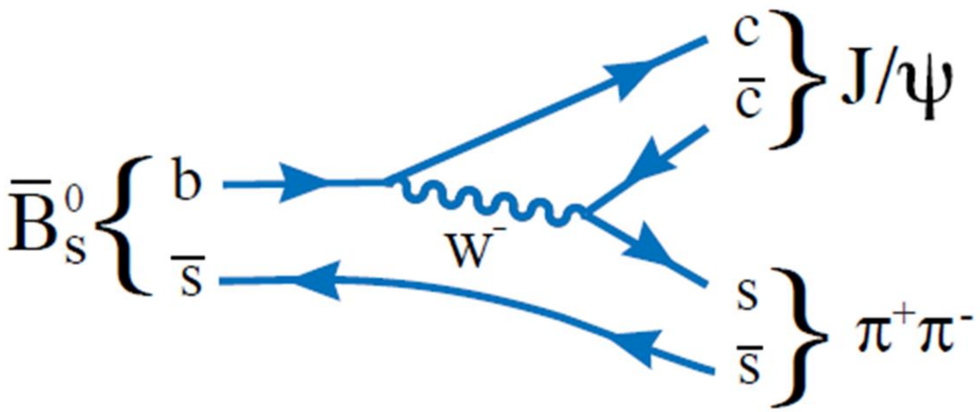
LHCb measurement

CERN-PH-EP-2013-024
LHCb-PAPER-2013-069
February 26, 2014

1402.6248

theoretical framework is ready
wait for data

Measurement of resonant and CP components in $\overline{B}^0_s \to J/\psi\pi^+\pi^-$ decays



I=0,
pure S wave

Fit fractions

Fit fractions (%) of contributing components for both solutions

Component	Solution I	Solution II
$f_0(980)$	$70.3 \pm 1.5^{+0.4}_{-5.1}$	$92.4 \pm 2.0^{+\ 0.8}_{-16.0}$
$f_0(1500)$	$10.1 \pm 0.8^{+1.1}_{-0.3}$	$9.1 \pm 0.9 \pm 0.3$
$f_0(1790)$	$2.4 \pm 0.4^{+5.0}_{-0.2}$	$0.9 \pm 0.3^{+2.5}_{-0.1}$
$f_2(1270)_0$	$0.36 \pm 0.07 \pm 0.03$	$0.42 \pm 0.07 \pm 0.04$
$f_2(1270)_\parallel$	$0.52 \pm 0.15^{+0.05}_{-0.02}$	$0.42 \pm 0.13^{+0.11}_{-0.02}$
$f_2(1270)_\perp$	$0.63 \pm 0.34^{+0.16}_{-0.08}$	$0.60 \pm 0.36^{+0.12}_{-0.09}$
$f_2'(1525)_0$	$0.51 \pm 0.09^{+0.05}_{-0.04}$	$0.52 \pm 0.09^{+0.05}_{-0.04}$
$f_2'(1525)_\parallel$	$0.06^{+0.13}_{-0.04} \pm 0.01$	$0.11^{+0.16+0.03}_{-0.07-0.04}$
$f_2'(1525)_\perp$	$0.26 \pm 0.18^{+0.06}_{-0.04}$	$0.26 \pm 0.22^{+0.06}_{-0.05}$
NR	-	$5.9 \pm 1.4^{+0.7}_{-4.6}$

S-wave 2-pion DAs

$$\Phi_{\pi\pi}^{S-wave} = \frac{1}{\sqrt{2N_c}}\left[\not{p}\Phi_{v\nu=-}^{I=0}(z,\zeta,w^2) + \omega\Phi_s^{I=0}(z,\zeta,w^2)\right.$$

$$\left. + \omega(\not{n}_+\not{n}_- - 1)\Phi_{t\nu=+}^{I=0}(z,\zeta,w^2)\right]$$

Gegenbauer moment to be determined

$$\phi_0 = \frac{9F_s(w^2)}{\sqrt{2N_c}} a_2^{I=0} z(1-z)(1-2z)$$

$$\phi_s = \frac{F_s(w^2)}{2\sqrt{2N_c}}, \quad \phi_\sigma = \frac{F_s(w^2)}{2\sqrt{2N_c}}(1-2z)$$

Flatte and BW models

to be determined

$$F_s^{s\bar{s}}(\omega^2) = \frac{c_1 m_{f_0(980)}^2 e^{i\theta_1}}{m_{f_0(980)}^2 - \omega^2 - i m_{f_0(980)}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK})}$$

pion time-like form factor

$$+ \frac{c_2 m_{f_0(1500)}^2 e^{i\theta_2}}{m_{f_0(1500)}^2 - \omega^2 - i m_{f_0(1500)}\Gamma_{f_0(1500)}(\omega^2)}$$

Flatte PLB, 1976

$$+ \frac{c_3 m_{f_0(1790)}^2 e^{i\theta_3}}{m_{f_0(1790)}^2 - \omega^2 - i m_{f_0(1790)}\Gamma_{f_0(1790)}(\omega^2)},$$

$$\rho_{\pi\pi} = \frac{2}{3}\sqrt{1 - \frac{4m_{\pi^\pm}^2}{\omega^2}} + \frac{1}{3}\sqrt{1 - \frac{4m_{\pi^0}^2}{\omega^2}} \qquad g_{\pi\pi} = 0.167\ \mathrm{GeV}$$

$$\rho_{KK} = \frac{1}{2}\sqrt{1 - \frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1 - \frac{4m_{K^0}^2}{\omega^2}} \qquad g_{KK} = 3.47 g_{\pi\pi}$$

nonresonant ~ 1/w^2 asymptotically

PQCD fit

Gegenbauer moment

$a_2^{I=0} = 0.2$

$$c_1 = 1.17, \quad c_2 = 0.12, \quad c_3 = 0.06,$$
$$\theta_1 = -\frac{\pi}{2}, \quad \theta_2 = \frac{\pi}{4}, \quad \theta_3 = 0.$$

interference among resonances

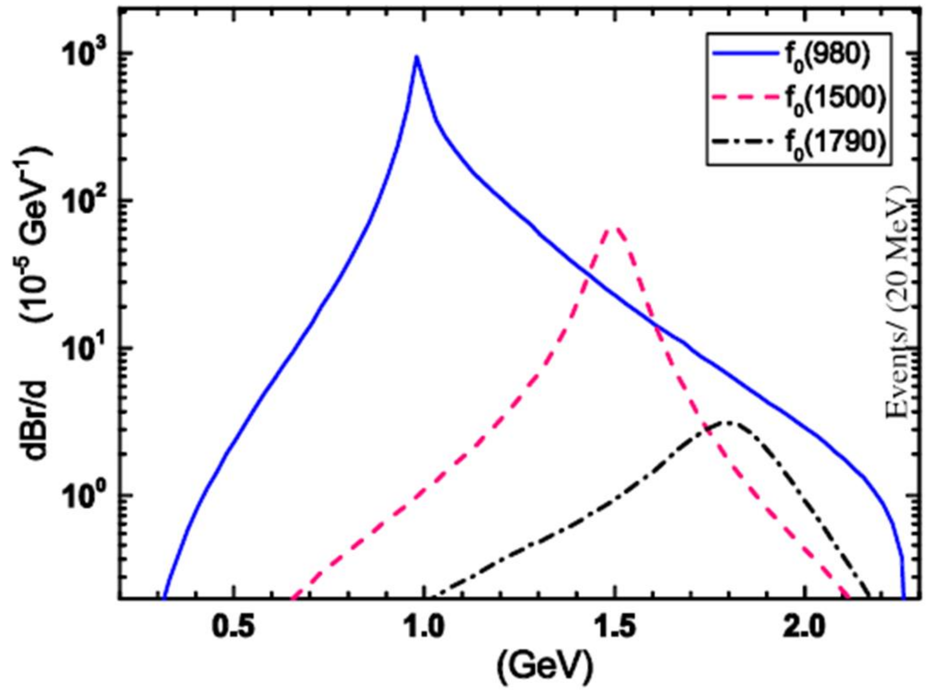
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(980)[f_0(980) \to \pi^+\pi^-])$$
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(1500)[f_0(1500) \to \pi^+\pi^-])$$
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(1790)[f_0(1790) \to \pi^+\pi^-])$$

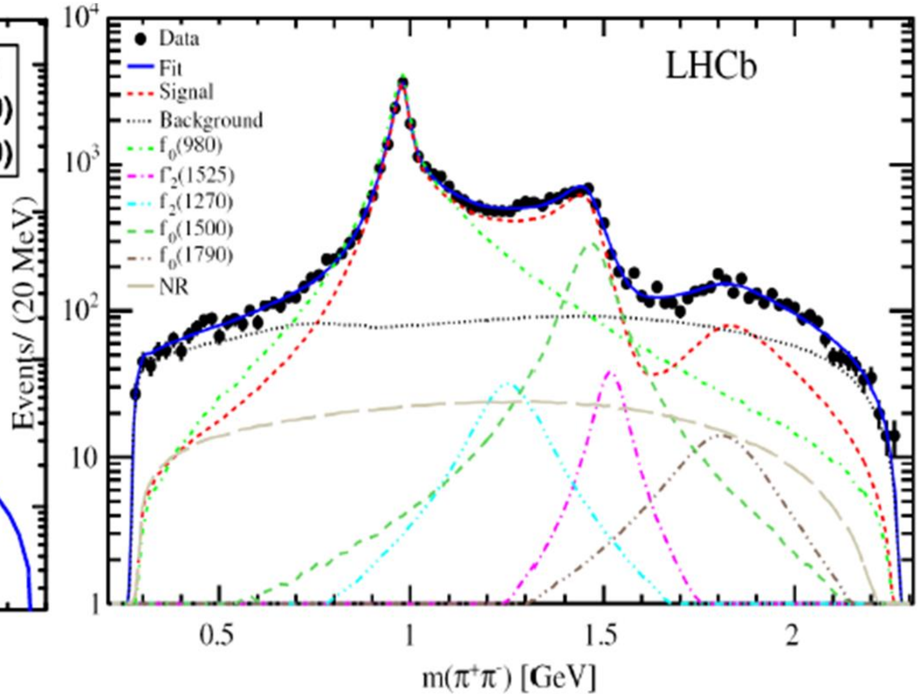
$$(1.33^{+0.51}_{-0.36}(\omega_{B_s})^{+0.19}_{-0.16}(a_2^{I=0})^{+0.03}_{-0.02}(m_c)) \times 10^{-4}\,, \quad 75.1\%$$
$$(1.77^{+0.53}_{-0.39}(\omega_{B_s})^{+0.30}_{-0.25}(a_2^{I=0}) \pm 0.02(m_c)) \times 10^{-5} \quad 10.0\%$$
$$(2.15^{+0.58}_{-0.49}(\omega_{B_s})^{+0.34}_{-0.32}(a_2^{I=0}) \pm 0.03(m_c)) \times 10^{-6} \quad 1.2\%$$

closer to Solution I of LHCb data

Comparison with data

$$B_s^0 \to J/\Psi\pi^+\pi^-$$





PQCD(NLO)

LHCb **(Sol 1)**